Demand-based structural change and balanced economic growth*

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Abstract

We analyze the equilibrium of a multi-sector exogenous growth model where the introduction of minimum consumption requirements drives structural change. We show that equilibrium dynamics simultaneously exhibit structural change and balanced growth of aggregate variables as is observed in US when the initial intensity of minimum consumption requirements is sufficiently small. This intensity is measured by the ratio between the aggregate value of the minimum consumption requirements and GDP and, therefore, it is inversely related with the level of economic development. Initially rich economies benefit from an initially low intensity of the minimum consumption requirements and, as a consequence, these economies end up exhibiting balanced growth of aggregate variables, while there is structural change. In contrast, initially poor economies suffer from an initially large intensity of the minimum consumption requirements, which makes the growth of the aggregate variables unbalanced during a very large period. These economies may never exhibit simultaneously balanced growth of aggregate variables and structural change.

JEL classification codes: O41, O47.

Keywords: structural change, non-homothetic preferences, balanced growth, speed of convergence.

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1. Introduction

The recent economic growth experience of US and some other developed countries is characterized by two different set of facts, which were illustrated by Kuznets (1957) and Kaldor (1961), respectively. The Kuznets facts are defined by the change in the sectoral shares of employment, which is a pattern observed in most economies. Figure 1 shows evidence of this long run trend in the US and it shows that during the period 1869 to 2005 labor moved from agriculture to manufactures and services. The Kaldor facts are observed in some developed economies during the last decades and are defined by the balanced growth of the aggregate variables. This balanced growth is identified by an almost constant interest rate and an almost constant value of the ratio of capital to GDP. Figure 2 shows that the time path of the ratio of capital to GDP in the US does not exhibit clear trends in the last decades. Therefore, during the last decades, some developed economies exhibit both balanced growth of aggregate variables and structural change. Recently, there is a growing interest in analyzing whether multisector growth models can simultaneously explain the Kaldor and Kuznets (K-K, henceforth) facts. In this paper, we contribute to this analysis.

[Insert Figures 1 and 2]

Most multisector-growth models cannot explain K-K facts when structural change is driven only by the accumulation of production factors. In these models, the equilibrium exhibits structural change and unbalanced growth during the transition, whereas it exhibits a constant sectoral composition and balanced growth in the long run. Therefore, these models cannot explain equilibrium dynamics along which aggregate variables exhibit an almost balanced growth path, while there is structural change. Recently, the growth literature has introduced additional factors driving structural change in order to explain both sets of facts. This literature has distinguished between models where structural change is driven by supply factors and models where it is driven by demand factors. On the one hand, supply factors are changes in relative prices that through a substitution effect cause structural change. These factors have been studied by Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Melck (2002), among many others. On the other hand, demand factors are related to income effects due to non-homothetic preferences that cause structural change in a growing economy. These factors have been studied by Kongsamut et al. (2001) (KRX, henceforth), Foellmi and Zweimüller (2008), among others.[1] Buera and Kaboski (2009), and Boppart (2014) combine both supply and demand factors to explain structural change.[2]

KRX introduce sector specific minimum consumption requirements in a multisector growth model. This model can explain the K-K facts when the intertemporal decision on consumption expenditures is driven by homothetic preferences, whereas the intratemporal decision on the allocation of expenditures among the different consumption goods is driven by non-homothetic preferences. The homotheticity of

[1] Echevarria (1997), Laitner (2000) and Caselli and Coleman (2001) are important papers analyzing structural change driven by non-homothetic preferences in a growing economy. However, the purpose of these papers is to explain the Kuznets facts and they do not discuss the Kaldor facts.

[2] Dennis and Iscan (2008) and Herrendorf et al. (2013) compare the performance of demand and supply factors to explain sectoral change.

preferences governing the intertermporal decision implies that aggregate variables converge to a BGP and the non-homotheticity of preferences governing the intratemporal decision causes structural change in a growing economy. As shown by KRX, these two conditions can be simultaneously satisfied when the aggregate value at market prices of the sector specific minimum consumption requirements is zero. Obviously, this is a knife-edge condition that requires strong assumptions on both preference parameters and technology. **KRX have already shown numerically that small deviations from this knife-edge condition are still consistent with an almost constant time path of the aggregate variables and structural change. We make a three-fold contribution to this analysis. First, we clarify the conditions making the equilibrium path exhibit almost constant time path of the aggregate variables and structural change. We show that this condition is related to the level of initial income. We use this result to explain that K-K facts are observed in economies that are initially developed. Second, we obtain interesting insights on the time path of GDP growth rate of those economies that do not satisfy the knife-edge condition. Third, we show that by deviating from the knife-edge condition the performance of the equilibria in explaining structural change improves and these equilibria still exhibit almost constant time path of the aggregate variables.**

We study a multisector growth model where structural change is driven by preferences that are non-homothetic due to the introduction of sector specific minimum consumption requirements. We show that, given initial conditions on capital intensity, there is a continuum of equilibrium paths indexed by the initial intensity of the minimum consumption requirements. This intensity is measured by the ratio between the aggregate value of the sector specific minimum consumption requirements and GDP. Obviously, in a growing economy, this intensity decreases and converges to zero. As a consequence, in the long run, preferences are homothetic and the economy converges asymptotically to the same BGP regardless of the initial intensity.

KRX, by assuming that the aggregate value of the minimum consumption requirements is zero, select one particular equilibrium path. This equilibrium is obtained when the initial intensity of the minimum consumption requirements is equal to zero. Along this equilibrium, convergence in the aggregate variables is faster than convergence in the sectoral composition. This implies that eventually aggregate variables exhibit a balanced growth path, while there is structural change. Note that this equilibrium explains the K-K facts. By using a continuity argument, we assert that these facts can also be explained by other equilibrium paths that are close enough to this equilibrium. These other equilibrium paths can be selected by assuming sufficiently small initial intensities of the minimum consumption requirements. Therefore, we argue that the necessary condition to explain the K-K facts is to assume a sufficiently small initial intensity of the minimum consumption requirements.[3] We prove this conclusion

[3]**The deviations from the knife-edge conditions considered by KRX are related to the value of the minimum consumption requirements. We contribute by showing that these deviations must be related to the ratio between the value of the minimum consumption requirements and the level of economic development and, therefore, the relevant variable is the intensity of the minimum consumption requirements.**

numerically. We simulate the transitional dynamics of economies that are differentiated only by the initial intensity of the minimum consumption requirement and we use two different criteria to show that there is a continuum of equilibrium paths satisfying the K-K facts. First, we show that if the aggregate value of the minimum consumption requirement is initially less than 25% of GDP, then the speed of convergence of variables characterizing the aggregate economy (interest rate and capital to GDP ratio) will be larger than the speed of convergence of those variables characterizing the sectoral composition (employment shares). This result implies that in these economies with a sufficiently small initial intensity of the minimum consumption requirements aggregate variables eventually exhibit balanced growth, while there is structural change. The second criterion is based on the value of the average annual growth rate of the variables in the last 65 years of the numerical simulations. We show than in economies with an initial value of the minimum consumption requirement smaller than 25% of the GDP, the annual growth rate of aggregate variables during this period is almost null, whereas the growth rate of the employment shares is clearly different from zero. Note that both criteria are consistent and suggest that those economies with a sufficiently small initial intensity of the minimum consumption requirements satisfy the K-K facts. In contrast, when the initially intensity is large, the growth of the aggregate variables is unbalanced during a large period of time and these equilibria may not satisfy the K-K facts.

We also study the performance of the numerical simulations in explaining structural change in the US during the period 1869-2005. We show that the equilibrium that provides the best fit is obtained when we assume that the initial intensity of the minimum consumption requirements is 25% of GDP. This result is an obvious consequence of the fact that in the model structural change is explained only by demand factors and, thus, in order to explain the patterns of structural change we need to assume that the US economy was suffering from a strong intensity of the minimum consumption requirements in the nineteen century.

In a last numerical exercise, we show that the convergence of the GDP growth rate crucially depends on the initial intensity of the minimum consumption requirements. When this intensity is sufficiently small, the growth rate decreases as capital accumulates, as in the neoclassical growth model. In contrast, when the initial intensity is large, the time path of the growth rate is hump-shaped. Interestingly, this pattern of convergence has been observed in some fast growing economies that were initially poor (South Korea, Taiwan, Japan).[4]

This paper then outlines the relevance of the initial intensity of the minimum consumption requirements in explaining the observed transitional dynamics. This variable is inversely related to the level of development. In initially rich economies, the intensity is low and hence these economies exhibit the K-K facts and neoclassical convergence of the growth rate. In contrast, in initially poor economies, the intensity of the minimum consumption requirements is large and growth is unbalanced during a longer period of time, implying that these economies may not exhibit the K-K facts. Moreover, the growth rate exhibits a hump-shaped transition in these economies.

[4]Easterly (1991), Christiano (1989) and, more recently, Steger (2001), Papageorgiou and Perez-Sebastian (2005) and Jeong and Yong Kim (2006) show that some fast growing economies exhibit a hump-shaped transition of the GDP growth rate.

In this paper structural change is driven only by income effects. However, Buera and Kaboski (2009), Herrendorf et al. (2013), and Boppart (2014) provide evidence showing that both relative price effects and income effects drive structural change. Kongsamut et al (1999) introduce relative price effects in his model of Stone-Geary preferences. However, in order to satisfy the K-K facts they introduce strong knife-edge conditions. By showing that the knife-edge conditions are not necessary to explain K-K facts, we show that relative price effects can be introduced in a model of minimum consumption requirements. In Section 6, we introduce relative price effects by assuming biased technological change in a version of the model where we do not impose any additional knife-edge condition. We show that the simulated economies are consistent with the K-K facts, implying that models with minimum consumption requirements can encompass both income and relative price effects in order to explain structural change and balanced growth. We then conclude that the introduction of sector specific minimum consumption requirements in multisector growth models provides a plausible explanation of the K-K facts.

The paper is organized as follows. Section 2 presents the model and Section 3 characterizes the equilibrium. Section 4 numerically simulates the equilibrium dynamics to investigate when the economy simultaneously satisfies the K-K facts. Section 5 analyses the performance of the numerical simulations. Section 6 extends the analysis to introduce biased technological change. Section 7 concludes the paper. The proof of stability is in Appendix A.

2. The Model

2.1. Firms

We consider an economy composed of m productive sectors. We interpret the sector m as the one producing manufactures that can be devoted to either consumption or investment, whereas all the other sectors produce a pure consumption good. We assume that each sector i produces by using the following Cobb-Douglas technology:[5]

$$Y_i = (s_i K)^{\alpha} (A_i u_i L)^{1-\alpha} = A_i u_i L (z_i)^{\alpha}, \tag{2.1}$$

where s_i is the share of total capital, K, employed in sector i; u_i is the share of total employment, L, in sector i; A_i measures the efficiency units of employment in sector i; α is the capital output elasticity; and $z_i = s_i K / A_i u_i L$ measures capital intensity in sector i. We assume that A_i grows at the exogenous growth rate γ, which is identical across sectors. This assumption implies that technological progress is unbiased and that the long run growth rate of GDP is equal to γ.

Finally, we assume perfect competition and perfect factor mobility across sectors, implying that each production factor is paid according to its marginal productivity and that wages, w, and the interest rate, r, are equal across sectors. This last assumption implies that

$$w = A_i p_i (1-\alpha)(z_i)^{\alpha}, \tag{2.2}$$

[5] For the sake of simplicity, time subindexes are not introduced.

and

$$r = p_i \alpha (z_i)^{\alpha-1} - \delta, \tag{2.3}$$

where $\delta \in [0,1]$ is the depreciation rate of capital and p_i is the relative price of the good produced in sector i in units of the good produced in sector m. Thus, the good produced in sector m is the numeraire and hence $p_m = 1$.

From using (2.2) and (2.3), we obtain that

$$z_i = \left(\frac{A_m}{A_i}\right) z_m, \tag{2.4}$$

and

$$p_i = \left(\frac{A_m}{A_i}\right)^{1-\alpha}. \tag{2.5}$$

Note that prices are constant as technological change is unbiased and capital output elasticity is the same across the different sectors. As a consequence, structural change is driven only by demand factors.[6]

2.2. Consumers

Let us consider an economy populated by an unique infinitely lived representative consumer. This consumer obtains income from capital and labor. This income is devoted to either consumption or investment. Therefore, the budget constraint is

$$rK + wL = \sum_{i=1}^{m} p_i c_i + \dot{K}, \tag{2.6}$$

where c_i is the amount consumed of good i. As follows from the budget constraint, the relative price of the investment good is one. This is a consequence of assuming that this good is produced in sector m and, as mentioned, the output of this sector is the numeraire. The representative consumer's utility function is

$$U = \int_0^\infty \left[\frac{\prod_{i=1}^{m} (c_i - \widetilde{c}_i)^{\theta_i(1-\sigma)}}{1-\sigma}\right] e^{-\rho t} dt, \tag{2.7}$$

where $\widetilde{c}_i$ is a preference parameter that can be interpreted as the minimum consumption requirement of good i; $\rho > 0$ is the subjective discount rate; $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution when $\widetilde{c}_i = 0$ for all i; and $\theta_i \in (0,1)$ provides the weights of the different consumption goods in the utility function. We assume that $\sum_{i=1}^{m} \theta_i = 1$. Note that this utility function is non-homothetic when $\widetilde{c}_i \neq 0$ for some i.

The representative consumer maximizes the utility function (2.7) subject to the budget constraint (2.6). By standard procedure, we find the first order conditions and rearrange them to summarize the necessary conditions for optimality in the following two conditions:[7]

$$U_i = p_i U_m, \tag{2.8}$$

[6] Alonso-Carrera, Caballé and Raurich (2014) study the transitional dynamics effects of changes in prices.

[7] The subindex in the utility function denote partial derivatives, implying that $U_i = \partial U / \partial c_i$ and $U_m = \partial U / \partial c_m$.

and

$$\frac{\dot{U}_m}{U_m} = \rho - r. \tag{2.9}$$

Using (2.7) and (2.8) we obtain

$$p_i\left(c_i - \widetilde{c}_i\right) = \left(\frac{\theta_i}{\theta_m}\right)\left(c_m - \widetilde{c}_m\right). \tag{2.10}$$

Equation (2.10) characterizes the intratemporal decision on the allocation of consumption expenditures among the different consumption goods. Let $E = \sum_{i=1}^{m} p_i c_i$ be the value of consumption expenditures and let $\widetilde{E} = \sum_{i=1}^{m} p_i \widetilde{c}_i$ be the aggregate value of the minimum consumption requirements. From using the definitions of E and $\widetilde{E}$, equation (2.10) can be rewritten to obtain

$$c_m - \widetilde{c}_m = \theta_m\left(E - \widetilde{E}\right).$$

Using this equation and (2.10), we obtain the expenditure shares in every sector

$$\frac{p_i c_i}{E} = \theta_i\left(\frac{E - \widetilde{E}}{E}\right) + \frac{p_i \widetilde{c}_i}{E}. \tag{2.11}$$

Log-differentiating equation (2.10) with respect to time and taking into account that prices are constant, we obtain

$$\frac{\dot{c}_i}{c_i - \widetilde{c}_i} = \frac{\dot{c}_m}{c_m - \widetilde{c}_m}. \tag{2.12}$$

We use (2.9) to obtain

$$\rho - r = \frac{\sum_{i=1}^{m} U_{mi}\dot{c}_i}{U_m}. \tag{2.13}$$

We use (2.7) and (2.12) to rewrite (2.13) as the following Euler equation:

$$\frac{\dot{E}}{E} = \Omega\left(r - \rho\right), \tag{2.14}$$

where $\Omega = \left(E - \widetilde{E}\right)/\sigma E$ is the intertemporal elasticity of substitution (IES, henceforth).

One can directly conclude from (2.11) that income effects drive structural change in expenditure shares when $\widetilde{c}_i \neq 0$ for some i. In addition, as follows from (2.14), balanced growth of aggregate variables requires a constant intertemporal elasticity of substitution. This elasticity is constant when $\widetilde{E}/E = 0$, which is satisfied asymptotically in a growing economy as E diverges to infinite. Obviously, in finite time this condition can only be satisfied if $\widetilde{E} = 0$. Following these arguments, KRX show that if $\widetilde{E} = 0$ and $\widetilde{c}_i \neq 0$ for some i then the equilibrium simultaneously exhibits balanced growth of aggregate variables and structural change and, therefore, the model can explain the K-K facts. However, this condition is a strong knife-edge condition as it requires both a strict relationship between preference and technological parameters and constant relative prices. **KRX argue that if $\widetilde{E}$ is close to zero then K-K facts are still satisfied as aggregate variables exhibit an almost balanced growth path, while there is substantial structural change.**[8] **In the following section, we clarify that the necessary condition to explain the K-K facts imposes a maximum value of the ratio between the value of the aggregate minimum consumption requirements and the level of income.**

3. The equilibrium

In order to characterize the equilibrium, we define the following transformed variables: $z = K/A_m L$, $e = E/Q$ and $\widetilde{e} = \widetilde{E}/Q$, where $Q = \sum_{i=1}^{m} p_i Y_i$ measures GDP. Note that the stock of aggregate capital per efficiency units of labor, z, is a measure of capital intensity and $\widetilde{e}$ measures the intensity of the minimum consumption requirements. Note also that $\widetilde{e}$ is inversely related to the level of income.

3.1. Market clearing

We proceed to obtain the market clearing conditions. Since sector m produces a commodity that can be used either as a consumption good or as an investment good, the market clearing condition for this sector is given by

$$Y_m = c_m + \dot{K} + \delta K.$$

By the contrary, since the other sectors only produce consumption goods, the market clearing condition in these sectors is $c_i = Y_i$, for all $i \neq m$, which can be rewritten as

$$u_i = \frac{c_i}{A_i L (z_i)^{\alpha}}. \tag{3.1}$$

Market clearing in the labor market implies that

$$\sum_{i=1}^{m} u_i = 1, \tag{3.2}$$

and in the capital market implies that $\sum_{i=1}^{m} s_i = 1$.

Using the definitions of z and z_i, we obtain that

$$z_i = \frac{s_i A_m z}{u_i A_i}. \tag{3.3}$$

From the last equation and (2.4), we obtain that $z_m u_i = z s_i$. From using this equation and the equilibrium conditions in the labor and capital markets, it follows that $z_m = z$ and $z_i = A_m z / A_i$. This last equation and (3.3) imply that $s_i = u_i$.

Finally, from the budget constraint we obtain that

$$Q = E + \dot{K} + \delta K. \tag{3.4}$$

Using (2.1) and (2.5), GDP can be rewritten as

$$Q = A_m L z^{\alpha}. \tag{3.5}$$

[8] **We follow Acemoglu and Guerrierie (2008) and we define an almost BGP as an equilibrium path along which the change in aggregate variables is almost null.**

3.2. Static equilibrium: sectoral composition

We proceed to obtain the employment shares as functions of the transformed variables: e, $\widetilde{e}$ and z. To this end, we first use (2.4), (2.5), (2.11), (3.1) and (3.5) to obtain the employment share in the consumption sectors

$$u_i = \theta_i \left(e - \widetilde{e}\right) + p_i \widetilde{v}_i, \text{ for all } i \neq m, \tag{3.6}$$

where $\widetilde{v}_i = \widetilde{c}_i / Q = \widetilde{c}_i \widetilde{e} / \widetilde{E}$. From using the equilibrium condition in the labor market, $u_m = 1 - \sum_{i=1}^{m-1} u_i$, we obtain the employment share in the manufacturing sector

$$u_m = 1 - \left(e - \widetilde{e}\right)\left(1 - \theta_m\right) - \widetilde{e} + \widetilde{v}_m. \tag{3.7}$$

3.3. Dynamic equilibrium: aggregate variables

We use the definition of e, (3.4) and (3.5) to obtain

$$\frac{\dot{K}}{K} = \left(1 - e\right) z^{\alpha - 1} - \delta.$$

We log-differentiate the definition of z and we use the previous equation to obtain the following differential equation governing the time path of z :

$$\frac{\dot{z}}{z} \equiv \kappa\left(e, z\right) = \left(1 - e\right) z^{\alpha - 1} - \delta - \gamma. \tag{3.8}$$

Next, the differential equation governing the time path of $\widetilde{e}$ is obtained from log-differentiating the definition of this variable and it is

$$\frac{\dot{\widetilde{e}}}{\widetilde{e}} = -\gamma - \alpha\kappa\left(e, z\right). \tag{3.9}$$

Finally, we log-differentiate the definition of the transformed variable e, and then we use (2.14) and the first order conditions from the firms' problem to obtain the following differential equation governing the time path of e :

$$\frac{\dot{e}}{e} = \left(\frac{\alpha z^{\alpha - 1} - \delta - \rho}{\sigma}\right)\left(\frac{e - \widetilde{e}}{e}\right) - \gamma - \alpha\kappa\left(e, z\right). \tag{3.10}$$

Given initial conditions on both z and $\widetilde{e}$, which we denote by z_0 and $\widetilde{e}_0$, respectively, the dynamic equilibrium is a path of $\{e, z, \widetilde{e}\}_{t=0}^{\infty}$ that solves the system of differential equations (3.8), (3.9) and (3.10) and satisfies the transversality condition $\lim_{t\to\infty} e^{-\rho t} U_m K = 0$.

The equilibrium is defined using one control variable, e, and two state variables, z and $\widetilde{e}$. Let $A_{m,0}$ be the initial value of A_m. It follows that $\widetilde{e}_0 = \widetilde{E} \big/ A_{m,0} L z_0^{\alpha}$ and thus the initial values z_0 and $\widetilde{e}_0$ can be chosen independently because of the initial value of A_m. Obviously, given z_0, the initial intensity of the minimum consumption requirements decreases as $A_{m,0}$ increases. Observe that the knife-edge condition introduced by KRX is $\widetilde{E} = 0$ and, thus, it implies $\widetilde{e} = 0$. Therefore, by assuming this knife-edge condition from the beginning, they reduce the dimensionality of the equilibrium.

Proposition 3.1. *There is an unique steady state and the value of the variables is* $\widetilde{e}^* = 0$,

$$z^* = \left(\frac{\sigma\gamma + \delta + \rho}{\alpha}\right)^{\frac{1}{\alpha-1}},$$

and

$$e^* = 1 - \frac{\alpha\left(\delta + \gamma\right)}{\sigma\gamma + \delta + \rho}.$$

Proposition 3.2. *The unique steady state is saddle path stable.*

Given that there are two state variables, saddle path stability implies that the dynamic equilibrium is a two-dimensional stable manifold. Therefore, given initial conditions on both state variables, there is an unique equilibrium path converging towards the steady state. However, given initial conditions on relative capital intensity, z_0, there is a continuum of equilibrium paths indexed by the initial value of the intensity of the minimum consumption requirements, $\widetilde{e}_0$. Taking this into account, we can reinterpret the knife-edge condition in KRX. This condition implies that $\widetilde{e}_0 = 0$. Therefore, this knife-edge condition is equivalent to select a particular equilibrium path of the two dimensional manifold. We know that the transitional dynamics of this equilibrium path eventually satisfies the K-K facts, implying that variables characterizing the aggregate economy converge faster than variables characterizing the sectoral composition. By a continuity argument, we argue that other equilibrium paths close enough will exhibit similar transitional dynamics and, therefore, they will also satisfy these two sets of facts. These equilibrium paths can be selected by assuming that the initial intensity of the minimum consumption requirements is sufficiently small, but different from zero. Note that this conclusion implies that the dynamic equilibrium exhibits K-K facts even though the knife-edge condition is not assumed. **It also implies that deviations from the knife-edge condition must be related to the level of development, as the relevant condition depends on the value of the initial intensity of the minimum consumption requirement. In the following section, we numerically prove this conjecture by measuring the maximum value of the initial intensity compatible with K-K facts and we show t**hat there are no qualitative differences between the equilibrium path when $\widetilde{e}_0 = 0$ and when $\widetilde{e}_0 \neq 0$.

4. Kuznets and Kaldor facts

We assume that there are three sectors: manufactures, agriculture and services. In order to calibrate the parameters, we use the independence of the time path of aggregate variables from the values of θ_i and $\widetilde{c}_i$. We therefore set the value of the rest of parameters to match targets for the aggregate variables. We assume that $\alpha = 0.35$, which implies that the aggregate labor income share equals 65%. The long run growth rate of GDP is $\gamma = 2\%$. We set $\delta = 5.6\%$ to obtain a ratio of investment to capital equal 7.6% in the long run. We set $\sigma = 2$, that implies a long run IES equal to 0.5, and $\rho = 0.014$ to replicate a long run interest rate equal to 5.4%. We normalize the level of GDP by assuming that $A_{m,0} = 1$ and $L = 1$. We assume that $z_0 = 0.75z^*$, whereas we consider the following values of $\widetilde{e}_0 : \{-0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.9\}$.[9] Note that we simulate seven economies that are differentiated only by the initial intensity of the minimum consumption requirement. Note also that the initial condition on the capital intensity implies that these economies must accumulate capital along the transition in order to converge. Using these parameters, we simulate the equilibrium and we obtain the time path of the aggregate variables. Finally, these time paths are used to estimate $\{\theta_i\}_{i=1}^2$ and $\{\widetilde{c}_i\}_{i=1}^2$ by ordinary least squares to fit the sectoral employment shares to actual US data between the years 1869 and 2005. More precisely, we use (2.5), (3.5) and (3.6) to rewrite the employment shares in the consumption sectors as

$$u_i = \theta_i \left(e - \widetilde{e}\right) + \left(\frac{A_m}{A_i}\right)^{1-\alpha} \left(\frac{\widetilde{c}_i}{A_m L z^{\alpha}}\right), \text{ for all } i \neq m.$$

Without loss of generality, we assume that $A_{i0} = 1$ for $i = 1, 2$ and the employment share can be rewritten as[10]

$$u_i = \theta_i \left(e - \widetilde{e}\right) + \widetilde{c}_i e^{-\gamma t} z^{-\alpha}.$$

Table 1 shows the estimates of $\{\theta_i\}_{i=1}^2$ and $\{\widetilde{c}_i\}_{i=1}^2$ obtained in the seven economies. Using these estimates, the value of θ_m is obtained from $\theta_m = 1 - \theta_1 - \theta_2$ and the value of $\widetilde{c}_m$ is obtained from $\widetilde{c}_m = \widetilde{e}_0 z_0^{\alpha} - \widetilde{c}_1 - \widetilde{c}_2$.

[Insert Table 1]

The estimated weights θ_i of the consumption goods in the utility function are quite similar to those obtained by Herrendorf et al. (2013). More precisely, we obtain a very close value for the weight θ_2 of services, whereas we obtain a slightly larger (smaller) value for the weight θ_1 (θ_m) of agriculture (manufactures). Table 1 also shows that in order to explain the patterns of structural change in employment in the US the minimum consumption requirements must satisfy the following ranking: $\widetilde{c}_1 > \widetilde{c}_m > \widetilde{c}_2$. In the simulated example, this ranking implies that the income elasticity of the demand of service goods is larger than one, whereas the income elasticity of the demand of agriculture goods is smaller than one. These elasticities explain the increase in the share of labor devoted to services and the reduction in the share of labor devoted to agriculture that we observe in the data. The table also shows that estimated values of the minimum consumption requirements deviates from the values that the literature suggested (see, e.g., Kongsamut et al., 2001; or Herrendorf et al., 2013). Firstly, we observe that the patterns of sectoral change in the US are in some cases compatible with positive values of $\widetilde{c}_2$. In particular, while we obtain a negative value of $\widetilde{c}_2$ for sufficiently small values of $\widetilde{e}_0$, the estimated value of $\widetilde{c}_2$ is positive when $\widetilde{e}_0$ is positive and large. As was explained by KRX, a negative value of $\widetilde{c}_2$ can be interpreted as home production of services. Secondly, the estimated value of $\widetilde{c}_m$ can be either positive or

[9] The conclusions obtained in the numerical analysis also hold if we had assumed that z_0 is a smaller fraction of z^*. Note also that if we had assumed that $z_0 = z^*$ then aggregate variables would not exhibit transitional dynamics when $\widetilde{e}_0 = 0$, whereas they would exhibit transitional dynamics when $\widetilde{e}_0 \neq 0$.

[10] If we had assumed that $A_{i0} \neq 1$ for $i = 1, 2$ then the labor shares would have been $u_i = \theta_i \left(e - \widetilde{e}\right) + \phi_i e^{-\gamma t} z^{-\alpha}$, where $\phi_i = A_{i0}^{\alpha-1} \widetilde{c}_i$. Thus, in this case, we would estimate ϕ_i instead of $\widetilde{c}_i$.

negative but it is always different from zero, which is in stark contrast with what was typically assumed by the related literature. Therefore, observe that all of the estimated minimum consumption requirements are strictly positive for a sufficiently large value of $\widetilde{e}_0$.

Figures 3, 4 and 5 illustrate the numerical simulations of the seven economies that are differentiated only by the initial intensity of the minimum consumption requirements. Figure 3 shows the time path of four aggregate variables: the ratio of capital to efficiency units of labor, the ratio of capital to GDP, the interest rate and the ratio of consumption expenditures to GDP. The equilibrium obtained by assuming $\widetilde{e}_0 = 0$ is the equilibrium obtained when we assume the knife-edge condition $\widetilde{E} = 0$ imposed by KRX. As follows from Figure 3, the transitional dynamics of this economy are qualitatively similar to those of economies obtained when the knife-edge condition is not assumed ($\widetilde{e}_0 \neq 0$). In particular, the different economies converge to the same long run equilibrium. This is a consequence of the fact that in a growing economy the intensity of the minimum consumption converges to zero, regardless of the initial condition, as shown in Figure 4. This implies that preferences are homothetic in the long run, which explains that these different economies converge to the same long run equilibrium, but obviously they do at different rates of convergence. Therefore, the relevant differences among these economies occur during the transition.

[Insert Figures 3 and 4]

Panel 4 in Figure 3 shows that economies with an initially large intensity of the consumption requirement devote a large fraction of GDP to consumption expenditures. As a consequence, investment in these economies is small in the initial periods, implying that both capital per unit of efficiency labor and the ratio of capital to GDP initially decrease (see Panels 1 and 2 in Figure 1). Obviously, in a growing economy that accumulates capital, this smaller capital accumulation causes a reduction in the speed of convergence of aggregate variables, implying that convergence occurs later.[11] This suggests that these economies with a large initial intensity of the minimum consumption requirement may not explain the K-K facts, as these facts require that variables characterizing the aggregate economy should converge before than those other variables characterizing the sectoral composition. Based on this argument, Tables 2 and 3 provide two different criteria in order to disentangle between simulated economies that satisfy the K-K facts and those other economies that may not satisfy these facts.

[Insert Table 2]

Table 2 uses as a criterion the comparison between the half life of aggregate variables (interest rate, ratio of capital to GDP and ratio of capital to efficiency units of labor) and the half life of those other variables characterizing the sectoral composition (employment shares). Half life is the number of years a variable takes to fill half of the initial distance to the steady state. Therefore, half life is a measure of the non-asymtotic speed of convergence. Obviously, K-K facts are satisfied when half life is

[11] Christiano (1989) introduces this argument to explain that minimum consumption requirements reduce the speed of convergence in a one-sector neoclassical growth model.

much smaller for aggregate variables than for those variables characterizing the sectoral composition. As follows from this table, when the initial intensity of the minimum consumption requirements is zero, half life is smaller for aggregate variables than for the employment shares. This implies that in this economy, obtained by assuming the knife-edge condition $\widetilde{E} = 0$, aggregate variables will exhibit an almost BGP, while there is structural change and, thus, this economy satisfies the K-K facts. Table 2 also shows that the half life of aggregate variables increases as the initial intensity of the minimum consumption requirements increases. However, for those economies with $\widetilde{e}_0 \leq 0.25$ half life of aggregate variables is still smaller than half life of the employment shares, which implies that equilibria in these cases also satisfy the two sets of aforementioned facts. In contrast, for the economies with $\widetilde{e}_0 \geq 0.5$ half life is larger for aggregate variables, which implies that equilibria in these cases do not explain the two sets of facts. Note that the results in this table provide numerical support to our conjecture that the equilibria obtained by assuming an initial value of the intensity of the minimum consumption requirement below a threshold eventually exhibit the K-K facts. We then conclude that these facts can be explained in a model of structural change driven by demand factors, even though the knife-edge condition $\widetilde{E} = 0$ in KRX is not introduced.

[Insert Table 3]

Following Acemoglu and Guerrierie (2008), in Table 3 we use the average annual growth rate in the last 65 years as a second criterion to test whether or not the simulated economies satisfy the K-K facts. The table compares the growth rates of aggregate variables with the growth rates of the employment shares. Satisfying the K-K facts requires the growth of aggregate variables to be almost null, whereas the growth of those variables characterizing sectoral composition should be significantly different from zero. As follows from the table, the growth rates of the employment shares are significantly different from zero in all the simulated economies. Obviously, this implies that in all these economies there is structural change during the last 65 years. In contrast, the growth rates of aggregate variables are almost null when $\widetilde{e}_0 \leq 0.25$, whereas they are larger than 0.1% when $\widetilde{e}_0 \geq 0.5$. These findings imply that K-K facts are explained when we assume a sufficiently small initial intensity of the minimum consumption. Note that this conclusion is consistent with the findings obtained in Table 2. We can then safely conclude that the necessary condition to explain the K-K facts is a sufficiently small but significantly different from zero initial intensity of the minimum consumption requirements. **This result has two relevant implications. First, it implies that K-K facts are compatible with huge deviations from the knife-edge condition introduced by KRX.**[12] **Second, the condition obtained implies that satisfying the K-K facts is related to the initial level of development. Those economies that are initially more developed benefit from a lower intensity of the minimum consumption requirement and, therefore, it is more likely that they satisfy K-K facts.**

As follows from Table 1, when $\widetilde{e}_0$ is sufficiently small so that the economy exhibits K-K facts, the estimated minimum consumption requirements are positive in agriculture

[12] **Note that our numerical findings imply that K-K facts are compatible with values of $\widetilde{E}$ up to 25% of GDP.**

and negative in the service sector. Note that these values of the minimum consumption requirements are consistent with the ones assumed by KRX.

5. Performance of the numerical simulations

In this section we analyze the goodness of our model in replicating the patterns of structural change observed in US, and how this performance depends on the intensity of the minimum consumption requirements. To this end, we first compare in Figure 5 the employment shares in the three sectors with actual US data. As follows from this comparison, the numerical simulations provide a very good fit when explaining the employment shares in agriculture and services.[13] Moreover, there are interesting differences in the performance of the different simulated economies. Table 4 shows the root men-squared error and coefficient of determination of different simulated economies for the three employment shares. We consider economies that are consistent with the Kaldor and Kuznets facts and, therefore, we constraint our analysis to economies satisfying $\widetilde{e}_0 \leq 0.25$. The best fit is obtained when the initial intensity of the minimum consumption requirements is large, $\widetilde{e}_0 = 0.25$. The performance of this numerical simulation is very good: the coefficient of determination in the agriculture sector is 0.94 and of the service sector is 0.76. This result is obtained because we assume that structural change is driven only by demand factors. This finding then implies that in order to explain the process of structural change in the US in the last 140 years it is necessary to assume that in the mid of the nineteen century the US suffered from a large intensity of the minimum consumption requirements.

[Insert Figure 5 and Table 4]

Figure 6 shows the time path of the growth rate of the GDP. Economies with an initially small intensity of the minimum consumption requirement exhibit the standard neoclassical convergence, explained by the diminishing returns to capital. In contrast, the time path of the growth rate of those economies with an initially positive and large intensity of the minimum consumption requirement exhibit a hump-shaped pattern. In these economies, the large initial intensity of the minimum consumption requirement prevents capital accumulation, which explains the initially low growth rates. As capital becomes scarce, the interest rate rises which explains the increasing path of the growth rate and of capital accumulation. Once capital becomes abundant, the diminishing returns to capital cause the reduction in the growth rates until convergence. This growth pattern is consistent with the observed growth patterns in some emerging economies (Japan, South Korea and Taiwan). In fact, this hump-shaped pattern has already been explained in the framework of a one sector growth model with non-homothetic preferences by Steger (2000). Therefore, the contribution of our paper to this literature studying the growth patterns is to show that the equilibrium dynamics of

[13] The coefficient of determination of the manufacturing sector is small. This is a consequence of both the small variability of the actual labor share in this sector and also of the calibration procedure that estimates the parameters to match only the structural change in the agriculture and service sectors. Given the labor market clearing condition (3.2), the simulated variation in the labor share of the manufacturing sector is also driven by the variation in the labor share of the other two sectors that was not explained by our estimations and simulations.

a multisector growth model with non-homothetic preferences are consistent with both the growth patterns and the observed process of structural change.[14]

[Insert Figure 6]

6. Biased technological change

The knife-edge condition considered by KRX implies that relative price should be constant. This is a strong assumption that is not supported by the data and, moreover, it excludes relative price effects driving structural change. Buera and Kaboski (2009), Herrendorf et al. (2013), and Boppart (2014) provide evidence showing that both relative price effects and income effects drive structural change. Kongsamut et al (1999) introduce relative price effects in his model of Stone-Geary preferences. However, in order to satisfy the K-K facts they impose strong knife-edge conditions involving the value of the parameters and of the initial stock of capital. By proving that knife-edge conditions are not necessary, we suggest that relative price effects can be easily incorporated in a model with Stone-Geary preferences. In this section, we show that a model of structural change with Stone-Geary preferences and relative price effects is consistent with K-K facts. To this end, we introduce biased technological change.

In this model, the variation in the relative price causes structural change because the price elasticity of the consumption demand is different from one. This elasticity is equal to $\left(c_i - (1-\theta_i)\,\widetilde{c}_i\right)/\,c_i$ for all good i. It is worth mentioning some relevant properties of this elasticity, which are in stark contrast with the properties obtained when the utility function is a homothetic CES function, as in Ngai and Pissarides (2007). First, even though the utility function is a Cobb-Douglas function in our model, the price elasticity is different from one due to the introduction of the minimum consumption requirement. Ngai and Pissarides (2004) need to impose non-unitary elasticity of substitution between consumption goods to obtain price elasticities different than one and, thus, structural change driven by the variation of prices. Second, the price elasticity is sectoral specific. Sectors have different minimum consumption requirements and, thus, a different price elasticity. Third, the price elasticity is not constant and converges to one. The last property implies that asymptotically price effects vanish, even though technological progress is permanently biased. As a consequence, none of the employment shares will converge asymptotically to zero. As mentioned, these results are in contrast with Ngai and Pissarides (2007), where at least one of the employment shares converges to zero, as in their paper the price elasticity is constant and different from one.

We first assume that A_i grows at the constant growth rate γ_i, which is sector specific. As it follows from (2.5), prices are no longer constant and they grow at the rate

$$\frac{\dot{p}_i}{p_i} = (1-\alpha)\left(\gamma_m - \gamma_i\right). \tag{6.1}$$

[14] Note that our analysis in Figure 6 cannot be used for deriving conclusions regarding the cross-country comparisons of the patterns of economic growth as we have assumed in all the simulated economies the same initial condition for the stock of capital in efficient units of labor.

As prices change, the Euler condition, obtained from combining (2.10) and (2.13), can be rewritten as

$$\frac{\dot{E}}{E} = \Omega\left(r-\rho\right) - \Omega\left(1-\sigma\right)\sum_{i\neq m}\theta_i\frac{\dot{p}_i}{p_i} + \sum_{i\neq m}\frac{\dot{p}_i\widetilde{c}_i}{E}. \tag{6.2}$$

The dynamic equations characterizing the transitional dynamics are modified as follows. First, from log-differentiating the definition of z and using (3.4) and (3.5), we obtain

$$\frac{\dot{z}}{z} \equiv \kappa\left(e,z\right) = \left(1-e\right)z^{\alpha-1} - \delta - \gamma_m. \tag{6.3}$$

Second, we log differentiate $\widetilde{e}$ to obtain:

$$\frac{\dot{\widetilde{e}}}{\widetilde{e}} = -\gamma - \alpha\kappa\left(e,z\right) + \frac{\sum_{i\neq m}\dot{p}_i\widetilde{c}_i}{\widetilde{E}}. \tag{6.4}$$

Finally, we log-differentiate the definition of the transformed variable e and we use (6.2) to obtain

$$\begin{aligned}\frac{\dot{e}}{e} \;&=\; \left(\frac{\alpha z^{\alpha-1}-\delta-\rho}{\sigma}\right)\left(\frac{e-\widetilde{e}}{e}\right) - \left(\frac{e-\widetilde{e}}{e}\right)\left(\frac{1-\sigma}{\sigma}\right)\sum_{i\neq m}\theta_i\frac{\dot{p}_i}{p_i} \\ &\quad + \sum_{i\neq m}\frac{\dot{p}_i\widetilde{c}_i}{E} - \gamma_m - \alpha\kappa\left(e,z\right).\end{aligned} \tag{6.5}$$

An equilibrium is a path $\left\{z, \widetilde{e}, e, \{p_i\}_{i=1}^{m-1}\right\}_{t=0}^{\infty}$ that given initial conditions z_0, $\widetilde{e}_0$, $\{A_{i,0}\}_{i=1}^{m}$ solve the system of differential equations (6.1), (6.3), (6.4), and (6.5) and satisfies (2.5) and the transversality condition $\lim_{t\to\infty} e^{-\rho t}U_m K = 0$.

We proceed to numerically solve this equilibrium. To this end, we assume the same three sectors than in Section 4 and we calibrate the parameters following the same strategy. In particular, as in Section 4, we set $\alpha = 35\%$, $\delta = 5.6\%$, $\sigma = 2$, $\rho = 0.014$, $\{A_{i,0}\}_{i=1}^{m} = 1$ and $L = 1$. Following the calibration in that section, we set $\gamma_m = 2\%$ to match the long run growth rate of GDP. We follow Ngai and Pissarides (2007) and we set $\gamma_a = 3.54\%$ and $\gamma_s = 0.46\%$ to match an average growth rate of relative prices in agriculture and services in the period 1929-1998 respectively equal to -1% and 1%. Finally, $\{\theta_i\}_{i=1}^{2}$ and $\{\widetilde{c}_i\}_{i=1}^{2}$ are set to fit the sectoral employment shares to actual US data between the years 1869 and 2005. Table 5 shows the values of these parameters in three simulated economies that are differentiated by the value of $\widetilde{e}_0$, that takes values $\{-0.25, 0, 0.25\}$. In these economies $z_0 = 0.75z^*$. Table 6 shows that the performance of these simulated economies in explaining the process of structural change is good. In this regard, when $\widetilde{e}_0 = 0.25$, the coefficient of determination of the employment share in the agriculture sector is 0.86 and in the service sector is 0.66. From the comparison between Tables 4 and 6, it follows that the performance of the simulations worsens when biased technological change is introduced. This is a consequence of two well-known problems associated with biased technological change. First, while we had assumed that TFP growth is constant, it has not been constant during this period (see Herrendorf et. al., 2014). Second, as explained by Ngai and Pissarides (2007), the assumed differences in TFP growth contribute to explain structural change when the price elasticity of the demand is lower than one. However, in the calibrated model, this elasticity is larger than one in the service sector, as the calibrated minimum consumption requirement is negative in this sector.

[Insert Tables 5 and 6]

Tables 7 and 8 show that the simulated economies satisfy the K-K facts. First, Table 7 shows that half life is smaller for aggregate variables than for the employment shares. Second, Table 8 shows that the average growth rate in the last 65 years of aggregate variables is almost null, whereas the average growth rate of the employment shares is clear different from zero. Therefore, the simulated economies satisfy K-K facts even when both income and relative price effects drive structural change.

[Insert Tables 7 and 8]

7. Concluding remarks

We have analyzed the equilibrium dynamics of a multi-sector growth model, where the introduction of sector specific minimum consumption requirements makes preferences be non-homothetic. The equilibrium is characterized by two state variables: the stock of capital and the intensity of the minimum consumption requirement. The knife-edge condition in KRX is equivalent to assume that the intensity of the minimum consumption requirement is zero. We use two different criteria to show numerically that this equilibrium path satisfies the K-K facts and we also show that other equilibrium paths selected by assuming sufficiently low values of the initial intensity of the minimum consumption requirements also satisfy these two sets of facts. We conclude from this numerical findings that the equilibrium exhibits K-K facts even if the initial intensity is clearly different from zero. In fact, we show that the simulations with the best performance are obtained when this intensity is initially 25%, which is a large value of the intensity. Our finding have two relevant implications.

First, Buera and Kaboski (2009), Herrendorf et al. (2013), and Boppart (2014) provide evidence showing that both relative price effects and income effects drive structural change. Therefore, models of structural change should be able to encompass both income and price effects. By proving that knife-edge conditions are not necessary, we show that relative price effects can be easily incorporated in a model with Stone-Geary preferences. Therefore, models with sector specific minimum consumption requirements provide an interesting benchmark to study structural change.

Second, we show that the initial intensity of the minimum consumption requirements plays a crucial role driving the transitional dynamics. This initial intensity is inversely related with the level of economic development. Initially rich economies benefit from an initially low intensity of the minimum consumption requirements and, as a consequence, aggregate variables exhibit balanced growth, whereas there is structural change. These economies replicate the K-K facts during a long period and the growth rate decreases with capital accumulation, as in the neoclassical one-sector growth model. In contrast, initially poor economies suffer from an initially large intensity of the minimum consumption requirements and, thus, the growth of the aggregate variables is unbalanced during a long period of time. In these economies, K-K facts are either satisfied during a small number of years or they may never be satisfied. Moreover, the convergence of aggregate variables in these initially poor economies is different from the convergence obtained in the neoclassical one sector growth model. In particular, the time path of the growth rate exhibits a hump-shaped transition.

References

[1] Acemoglu, D. , Guerrieri, V. (2008). Capital deepening and non-balanced economic growth. *Journal of Political Economy*, 116(3), 467-498.

[2] Alonso-Carrera, J., Caballé, J. and Raurich, X. (2014). Sectoral Composition and Macroeconomic Dynamics. Forthcoming in *BE J. Macroecon. Advances.*

[3] Boppart, T. (2014). Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences. Forthcoming in *Economometrica*.

[4] Buera, F. and Kaboski, J. (2009). Can traditional theories of structural change fit the data? *Journal of the European Economic Association,* 7, 469-477.

[5] Caselli, F. and Coleman, J.W II. (2001).The U.S. structural transformation and regional convergence: A reinterpretation. *Journal of Political Economy,* 109, 584–616.

[6] Christiano, L (1989). Understanding Japan's Saving Rate: The Reconstruction Hypothesis. Federal Reserve Bank of Minneapolis Quarterly Review 13, 2.

[7] Dennis, B. and Iscan, T. (2009). Engel versus Baumol: accounting for sectoral change using two centuries of US data. *Explorations in Economic History*, 46, 186-202.

[8] Easterly, W (1991). Economic Stagnation, Fixed Factors, and Policy Thresholds, World Bank working paper, 795.

[9] Echevarría, C. (1997). Changes in sectoral composition associated with economic growth. *International Economic Review,* 38, 431-452.

[10] Foellmi, R. and Zweimüller, J. (2008). Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. *Journal of Monetary Economics*, 55(7):1317-1328.

[11] Herrendorf, B., Rogerson, R. and Valentinyi, Á. (2013). Two Perspectives on Preferences and Structural Transformation, *American Ecomic Review,* 103, 2752-2789.

[12] Herrendorf, B. ., Rogerson, R. and Valentinyi, Á. (2014). Growth and Structural Transformation, Handbook of Economic Growth, in: Handbook of Economic Growth, edition 1, volume 2, chapter 6, pages 855-941 Elsevier.

[13] Jeong, H. and Kim, Y. (2006). S-shaped Transition and Catapult Effects, unpublished manuscript.

[14] Kaldor, N. (1961). Capital accumulation and economic growth, in F.A. Lutz and D.C. Hague (eds.), The theory of capital. New York:St. Martin's Press, 177-222.

[15] Kongsamut, P., Rebelo, S., Xie, D. (1999). Beyond balanced growth. Working Paper, Northwestern University.

[16] Kongsamut, P., Rebelo, S., Xie, D. (2001). Beyond balanced growth. *Review of Economic Studies,* 68, 869-882.

[17] Kuznets, S. (1957). Quantitative aspects of the economic growth of nations: II. *Economic Development and Cultural Change*, Supplemanet to Vol. 5, 3-111.

[18] Laitner, J. (2000). Structural change and economic growth. *Review of Economic Studies,* 67, 545-561.

[19] Melck, J. (2002). Structural Change and Generalized Balanced Growth. *Journal of Economics*, 77, 241-266.

[20] Ngai, R., Pissarides, C. (2007). Structural change in a multisector model of growth. *American Economic Review,* 97, 429-443.

[21] Papageorgiou, C. and Perez-Sebastian, F. (2006). Dynamics in a non-scale R&D growth model with human capital: Explaining the Japanese and South Korean development experiences, *Journal of Economic Dynamics and Control*, 30, 901–930.

[22] Steger, T. (2000). Economic growth with subsistence consumption. *Journal of Development Economics,* 62, 343–361.

[23] Steger, T.M. (2001). Stylized facts of economic growth in developing countries, Discussion Paper 08/2001.

A. Appendix

Proof of Proposition 3.3

From equation (3.8), we obtain $\frac{\partial \dot{z}}{\partial \widetilde{e}} = z\kappa_{\widetilde{e}} = 0$, $\frac{\partial \dot{z}}{\partial e} = z\kappa_e = -z^{\alpha} < 0$ and[15]

$$\frac{\partial \dot{z}}{\partial z} = z\kappa_z = (\alpha - 1)(\delta + \gamma_m) < 0.$$

From equation (3.10), we obtain $\frac{\partial \dot{e}}{\partial \widetilde{e}} = -\gamma$, $\frac{\partial \dot{e}}{\partial e} = -e\alpha\kappa_e = e\alpha z^{\alpha-1} > 0$, and

$$\frac{\partial \dot{e}}{\partial z} = e\left(\frac{\alpha(\alpha-1) z^{\alpha-2}}{\sigma} - \alpha\kappa_z\right).$$

From equation (3.9), we obtain $\frac{\partial \dot{\widetilde{e}}}{\partial z} = 0$, $\frac{\partial \dot{\widetilde{e}}}{\partial e} = 0$, and $\frac{\partial \dot{\widetilde{e}}}{\partial \widetilde{e}} = -\gamma < 0$. The Jacobian matrix is

$$J = \begin{pmatrix} \frac{\partial \dot{z}}{\partial z} - \lambda & \frac{\partial \dot{z}}{\partial e} & 0 \\ \frac{\partial \dot{e}}{\partial z} & \frac{\partial \dot{e}}{\partial e} - \lambda & \frac{\partial \dot{e}}{\partial \widetilde{e}} \\ 0 & 0 & \frac{\partial \dot{\widetilde{e}}}{\partial \widetilde{e}} - \lambda \end{pmatrix},$$

and the characteristic polynomial is

$$P(J) = \left(\frac{\partial \dot{\widetilde{e}}}{\partial \widetilde{e}} - \lambda\right)\left[\left(\frac{\partial \dot{e}}{\partial e} - \lambda\right)\left(\frac{\partial \dot{z}}{\partial z} - \lambda\right) - \frac{\partial \dot{z}}{\partial e}\frac{\partial \dot{e}}{\partial z}\right].$$

The roots are $\lambda_1 = \frac{\partial \dot{\widetilde{e}}}{\partial \widetilde{e}} = -\gamma < 0$, and the solutions of

$$\lambda^2 - \lambda\left(\frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{e}}{\partial e}\right) + \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{e}}{\partial e} - \frac{\partial \dot{z}}{\partial e}\frac{\partial \dot{e}}{\partial z} = 0,$$

where

$$\frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{e}}{\partial e} - \frac{\partial \dot{z}}{\partial e}\frac{\partial \dot{e}}{\partial z} = -e\alpha\kappa_e z\kappa_z - z\kappa_e e\left(\frac{\alpha(\alpha-1) z^{\alpha-2}}{\sigma} - \alpha\kappa_z\right) = \frac{e\alpha(\alpha-1) z^{(\alpha-1)2}}{\sigma} < 0.$$

This term being negative implies that $\lambda_2 > 0$ and $\lambda_3 < 0$.

[15] We use the following notation for partial derivatives $\kappa_{\widetilde{e}} = \frac{\partial \kappa}{\partial \widetilde{e}}$, $\kappa_e = \frac{\partial \kappa}{\partial e}$ and $\kappa_z = \frac{\partial \kappa}{\partial z}$.

B. Tables and Figures

Table 1. Parameters estimated by OLS

$\widetilde{e}_0$	-0.5	-0.25	0	0.25	0.5	0.75	0.9
θ_1	0.0070	0.0072	0.0076	0.0083	0.0097	0.0126	0.0172
θ_2	0.8917	0.8941	0.8972	0.9012	0.9065	0.9142	0.9219
θ_m	0.1013	0.0987	0.0952	0.0905	0.0838	0.0732	0.0609
$\widetilde{c}_1$	1.1269	1.1089	1.0860	1.0560	1.0138	0.9444	0.8570
$\widetilde{c}_2$	-1.8438	-1.4896	-1.1355	-0.7804	-0.4214	-0.0491	0.2039
$\widetilde{c}_m$	-0.1096	-0.0325	0.0495	0.1376	0.2341	0.3444	0.4269

Table 2. Half life

$\widetilde{e}_0$	r	$\frac{K}{Q}$	z	u_1	u_2	u_m
-0.5	3	4	4	29	26	8
-0.25	5	5	5	30	27	10
0	8	9	10	31	30	17
0.15	16	18	19	32	31	27
0.25	27	30	31	32	33	36
0.5	58	62	63	35	37	65
0.75	78	82	83	38	42	88
0.9	87	91	93	43	50	97

Table 3. Average annual growth rate in the last 65 years

$\widetilde{e}_0$	r	$\frac{K}{Q}$	z	u_1	u_2	u_m
-0.5	0.11%	-0.05%	-0.08%	-1.81%	0.27%	-0.007%
-0.25	0.068%	-0.028%	-0.04%	-1.81%	0.27%	-0.01%
0	-0.0021%	0.0010%	0.0016%	-1.8269%	0.2759%	-0.0225%
0.15	-0.0444%	0.0221%	0.0340%	-1.8285%	0.2779%	-0.0296%
0.25	-0.0758%	0.038%	0.0585%	-1.8285%	0.2795%	-0.0353%
0.5	-0.1705%	0.0877%	0.1349%	-1.8215%	0.2846%	-0.0552%
0.75	-0.3080%	0.1634%	0.2515%	-1.7881%	0.2915%	-0.0903%
0.9	-0.4610%	0.2520%	0.3880%	-1.7161%	0.2974%	-0.1364%

Table 4. Performance of the simulations

	Agriculture		Services		Manufactures	
$\widetilde{e}_0$	$RMSE$	R^2	$RMSE$	R^2	$RMSE$	R^2
-0.5	0.0431	0.9282	0.0998	0.7274	0.0667	-0.0659
-0.25	0.0423	0.9308	0.0981	0.7363	0.0659	-0.0402
0	0.0412	0.9342	0.0962	0.7469	0.0649	-0.0082
0.25	0.0398	0.9386	0.0936	0.76	0.0635	0.0333

Note: The root mean-squared error ($RMSE$) and the coefficient of determination (R^2) are obtained from regressing the HP-filtered trend of actual employment shares on the simulated employment shares for each value of $\widetilde{e}_0$.

Table 5. Parameters. Biased technological change

$\widetilde{e}_0$	-0.25	0	0.25
θ_1	0.11	0.06	0.09
θ_2	0.88	0.94	0.91
θ_m	0.01	0.001	0.001
$\widetilde{c}_1$	0.80	1.20	1.085
$\widetilde{c}_2$	-1.14	-0.90	-0.80
$\widetilde{c}_m$	0.77	0.13	0.148

Table 6. Performance of the simulations. Biased technological change

	Agriculture		Services		Manufactures	
$\widetilde{e}_0$	$RMSE$	R^2	$RMSE$	R^2	$RMSE$	R^2
-0.25	0.07	0.77	0.08	0.81	0.077	-0.43
0	0.063	0.84	0.11	0.69	0.084	-0.87
0.25	0.063	0.86	0.11	0.66	0.0728	-0.27

Table 7. Half life. Biased technological change

$\widetilde{e}_0$	r	$\frac{K}{Q}$	z	u_1	u_2
-0.25	3	4	4	21	43
0	9	10	10	21	57
0.25	26	27	28	23	37

Table 8. Average annual growth rate in the last 65 years. Biased technological change

$\widetilde{e}_0$	r	$\frac{K}{Q}$	z	u_1	u_2	u_m
-0.25	-0.14%	0.06%	0.09%	-0.56%	0.28%	-0.31%
0	-0.0032%	0.0014%	0.0022%	-1.15%	0.31%	-0.18%
0.25	-0.0129%	0.0058%	0.009%	-0.84%	0.28%	-0.16%

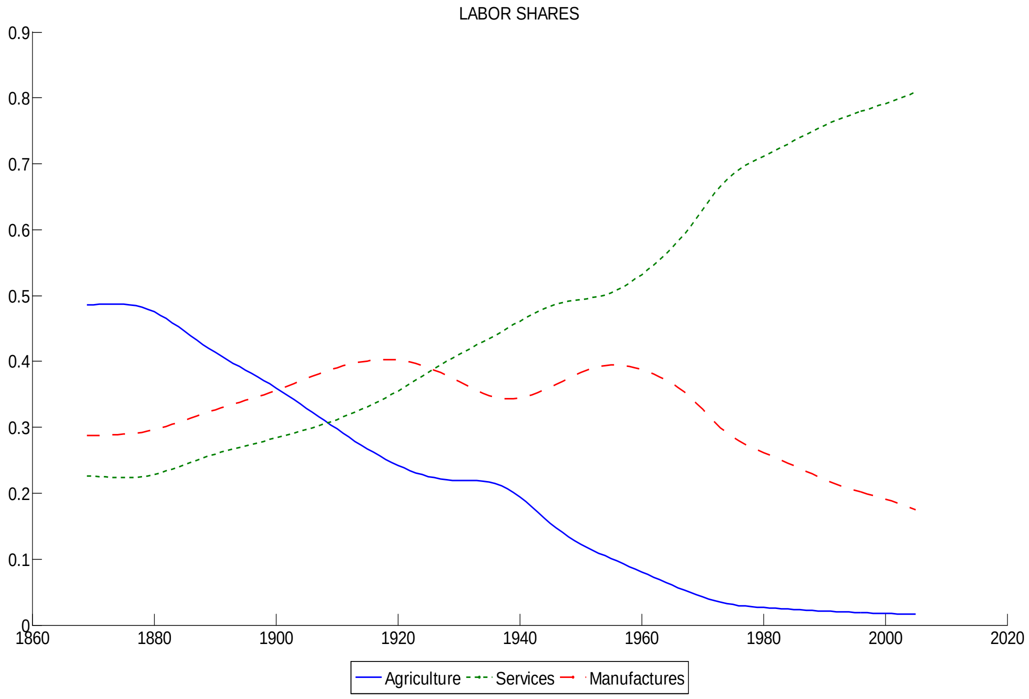

Figure1. Labor shares in the US. Source: Historical statistics of the U.S.

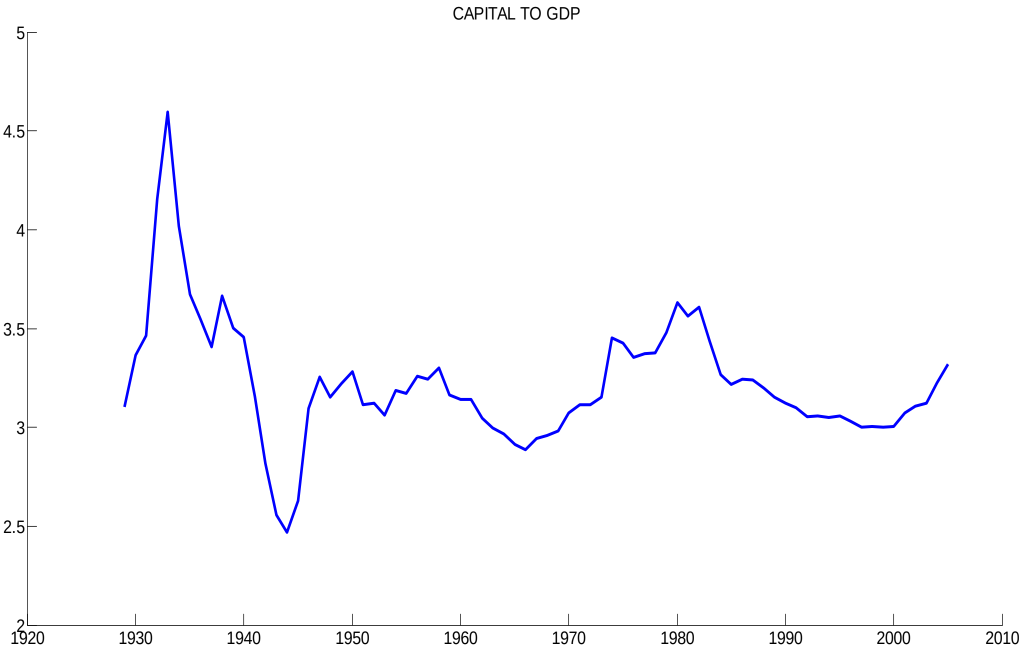

Figure2. Ratio of capital to GDP in the US. Source: U.S. Bureau of Economic Analysis.

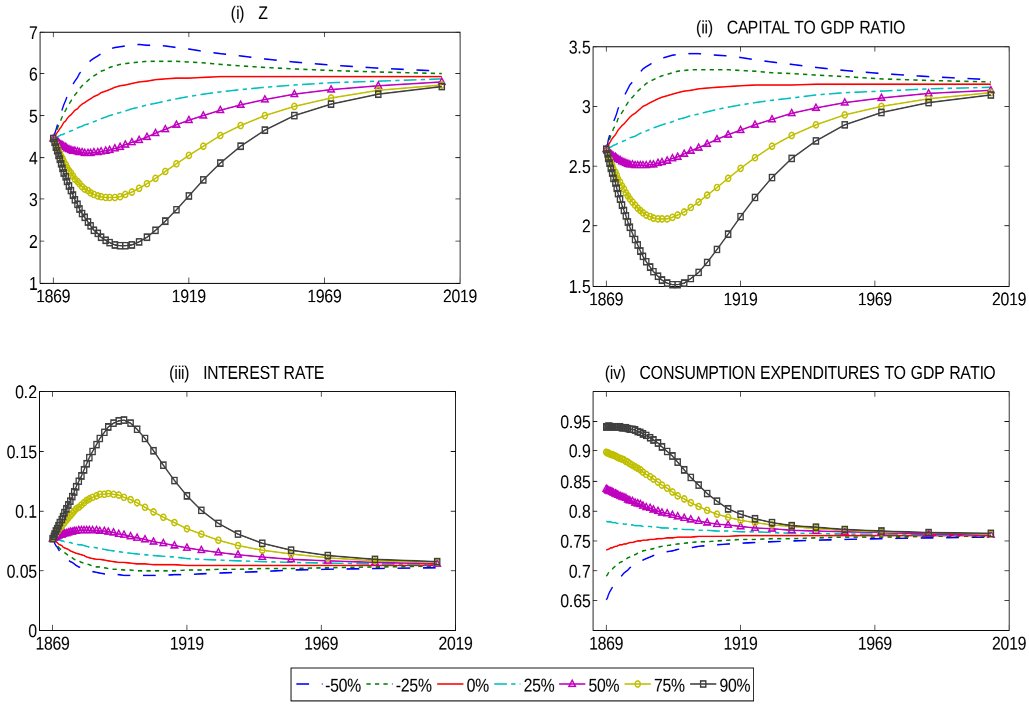

Figure3. Time path of the aggregate variables.

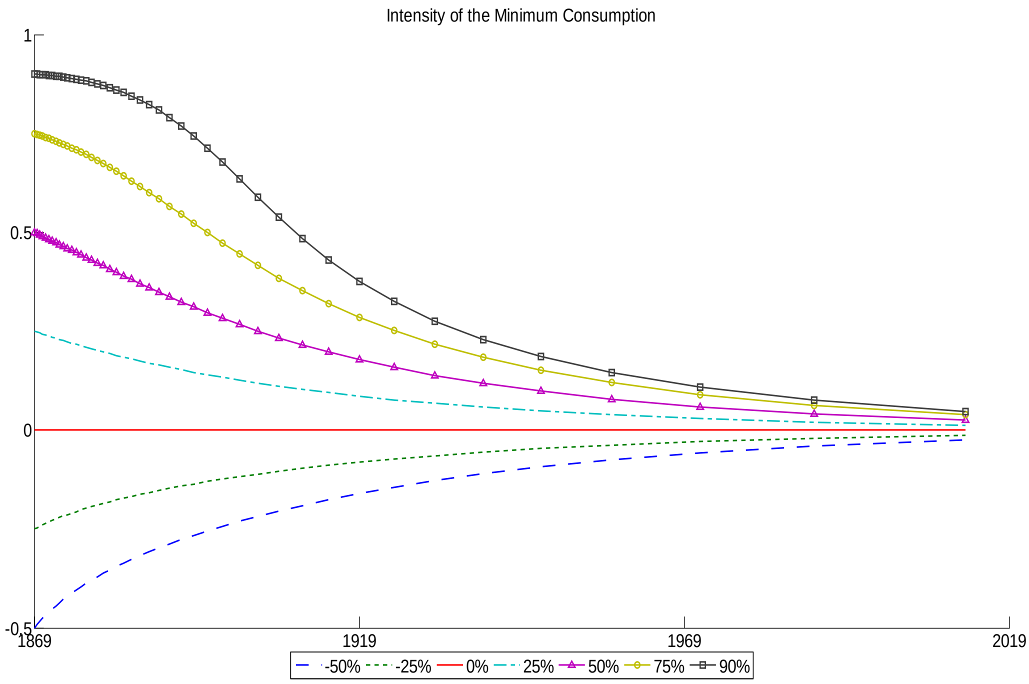

Figure4. Ratio between the value of the minimum consumption and GDP.

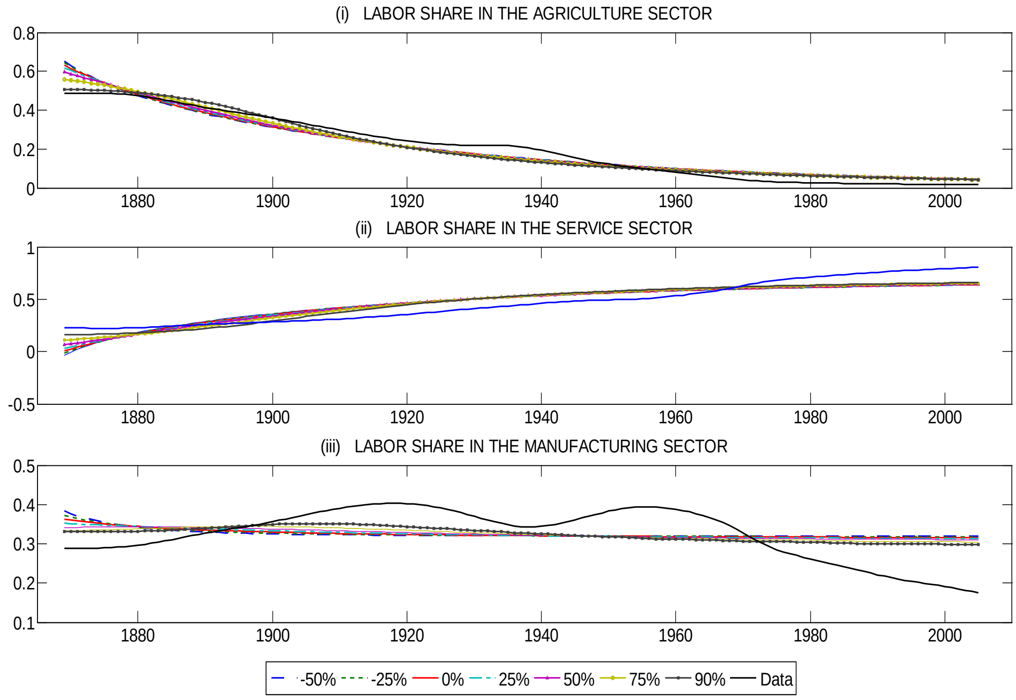

Figure5. Time path of the labor shares.

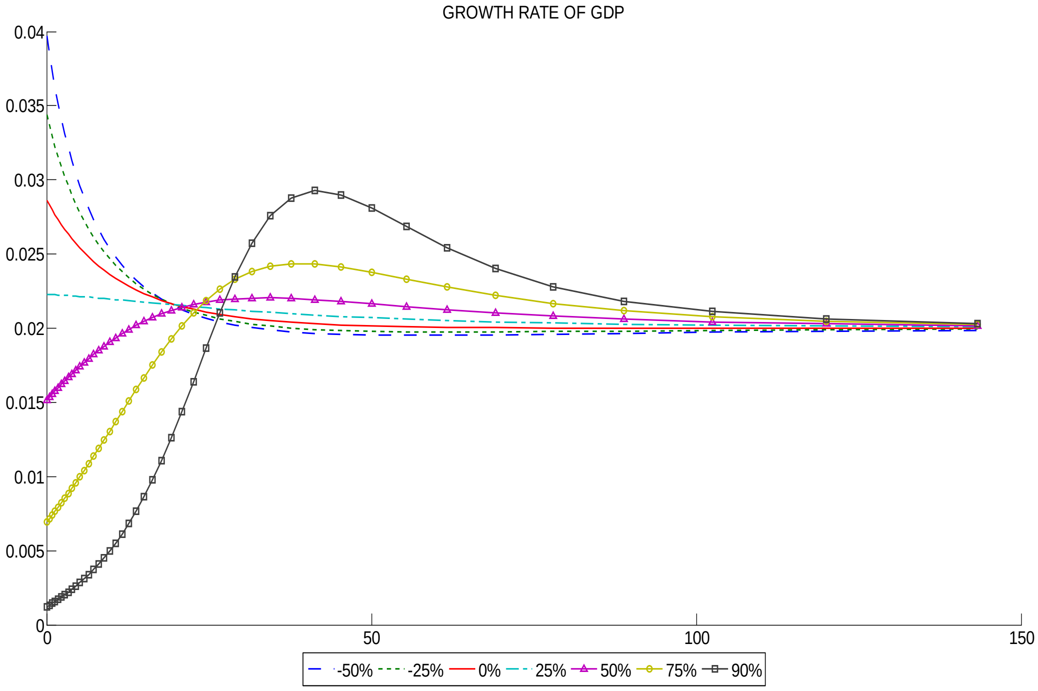

Figure6. Time path of the GDP growth rate.